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REPRINT

TP72-1

F. H. Raab, "Radio frequency pulsewidth modulation," ***IEEE Trans. Commun*****., vol. COM-21, no. 8, pp. 958 - 966, Aug. 1973.**

where $A_U + \alpha = R \cos\theta$ and $A_L + \beta = R \sin\theta$. Letting $\cos^2\theta = (1 - \cos 2\theta)/2$, integrating with respect to θ, and setting $A_U = A_L = A$, we obtain

$$\overline{w}^2 = \frac{1}{2N}\int_0^\infty R \exp\left[-\frac{R^2 + 2A^2}{2N}\right] \cdot I_0[R\sqrt{2A^2}/N]\, dR$$
$$= \tfrac{1}{2}. \tag{B-2b}$$

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Radio Frequency Pulsewidth Modulation

FREDERICK H. RAAB

***Abstract*—The width of a train of square pulses can be varied to produce a modulated carrier at the pulse repetition frequency. When the pulse train is generated by switching (class D) transistors, high-efficiency operation is possible. The efficiency of this type of amplifier can be significantly higher than that of conventional pulsewidth modulation amplifiers, since the switching rate is reduced. In addition, the spectrum of a bipolar pulse train so modulated has the highly desirable property of all spurious products being band limited near the odd harmonics of the carrier.**

I. INTRODUCTION

Class D or "switched mode" can be used to make a high-efficiency amplifier. Ideally, current flows through a device (transistor or tube) only when the voltage across it is zero, and no power is dissipated. The increased efficiency (compared to classes A, B, or C) results in not only a decrease in the dc input power, but also a decrease in the dissipated power. This allows the use of smaller devices and a smaller heat sink.

Class D has been used for several years to make highly efficient audio and servo amplifiers. This "conventional" pulsewidth modulation (PWM) amplifier generates a constant-frequency train of pulses whose widths are porportional to the desired output. A low-pass filter averages the signal to produce the desired output. Modulation of the pulsewidth to vary the dc component of the pulse train linearly with the input produces nonlinear modulation of the switching frequency component and its harmonics, generating infinite bandwidth spurious products. As a consequence, the switching frequency must be at least twice (generally five or more times) the highest audio frequency to be amplified, so that spurious products in the output of the low-pass filter are of acceptably low levels. An amplifier of this type is used as the plate modulator of a 100-kW transmitter [1]. Recently, Attwood [2] has extended this technique to 2 MHz to make a 90 percent efficient RF amplifier by switching at 10 MHz.

Paper approved by the Radio Communication Committee of the IEEE Communications Society for publication without oral presentation. Manuscript received January 19, 1973. This work is based in part on a portion of a dissertation by F. H. Raab in partial fulfillment of the requirements for the Ph.D. degree at Iowa State University.

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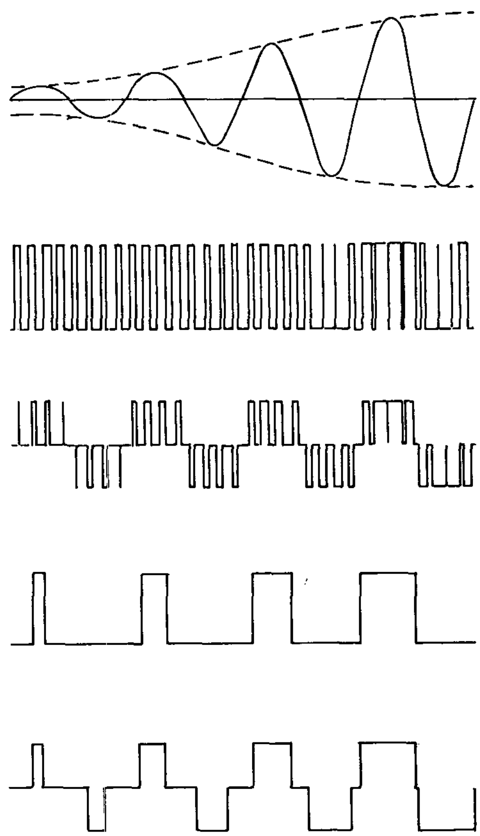


Fig. 1. Types of pulsewidth modulation.

Class D has also been used to generate radio frequency signals by switching at the RF frequency [3]–[5]. A tuned circuit replaces the low-pass filter and the amplitude of the output depends on the collector supply voltage. A high-efficiency modulated or linear amplifier can be obtained by using a class-D audio amplifier to vary the collector voltage of a class-D RF amplifier [6].

If the width of the pulses in a class-D RF amplifier is varied, then the amplitude of the RF carrier being generated is also varied. This modulation process was apparently invented by Besslich [7]. Either monopolar or bipolar pulse trains can be used; pulse trains for an AM signal are shown in Fig. 1.

RF pulsewidth modulation can have advantages over either conventional PWM or a combination of conventional PWM modulator and a class-D RF amplifier. In the former case, the switching rate is lowered to the carrier frequency and spurious products are concentrated around the harmonics of the carrier frequency. In the latter case, the separate modulator circuitry and its filters are eliminated. However, RF PWM pulse generation and amplification circuits must be faster than those of audio frequency PWM amplifiers.

The fundamental component of a pulse train is proportional to the sine of the pulsewidth. Therefore, it is necessary to make the pulsewidth vary as the inverse sine of the modulating signal. This can be accomplished by a simple comparator.

Let the modulating signal $x(t)$ be normalized such that

$$0 \leqslant x(t) \leqslant 1 \tag{1.1}$$

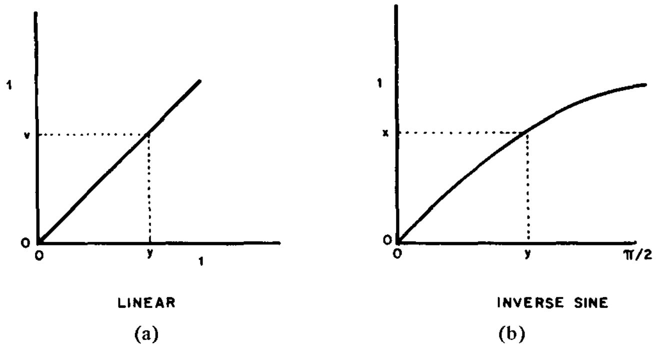


Fig. 2. Relationships between modulating signal and pulsewidth.

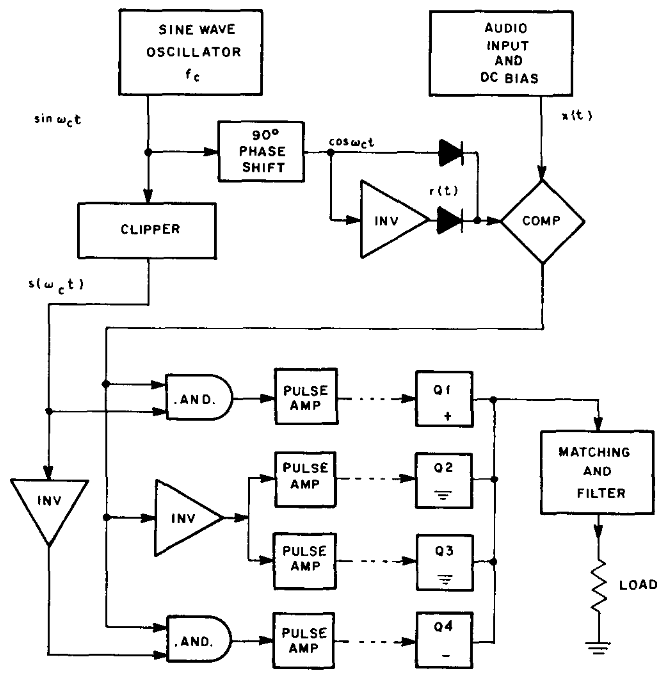


Fig. 3. Class-D AM transmitter. Q_1-Q_4 are transistors such as in the circuit of Fig. 8.

and the pulse half-width $y(t)$ in radians be such that

$$0 \leqslant y(t) \leqslant \pi/2. \tag{1.2}$$

Fig. 2 illustrates basic linear and inverse sine relationships. For a given position of x on the vertical axis, the position of y on the horizontal axis is such that

$$y = \arcsin x. \tag{1.3}$$

If a reference wave $r(t)$ is composed of pieces having the shape of the curve of Fig. 2(a), it will have the form

$$r(t) = |\cos \omega_c t|, \tag{1.4}$$

where ω_c represents the carrier frequency. Pulses generated when $x(t)$ is larger than $v(t)$ will have a half-width proportional to the inverse sine of $x(t)$.

An amplitude-modulated transmitter utilizing this principle must employ appropriate logic circuits to produce pulses of proper polarity at the output (Fig. 3). A simple means of doing this is by clipping the input RF carrier ($\sin \omega_c t$) to get a +1/−1 square wave $s(\omega_c t)$. The reference wave is generated by phase shifting the carrier and full-wave rectifying it. Pulses are generated by comparison of the input and reference waves (Fig. 4), but are all the same polarity. The output pulses then assume the polarity of the clipped signal and the width of the comparator output pulses. The pulses are then amplified to the desired level and the fundamental coupled to the output by appropriate matching and filtering. The audio bandwidth is essentially limited only by the carrier frequency (and spurious products) and frequency response is essentially flat.

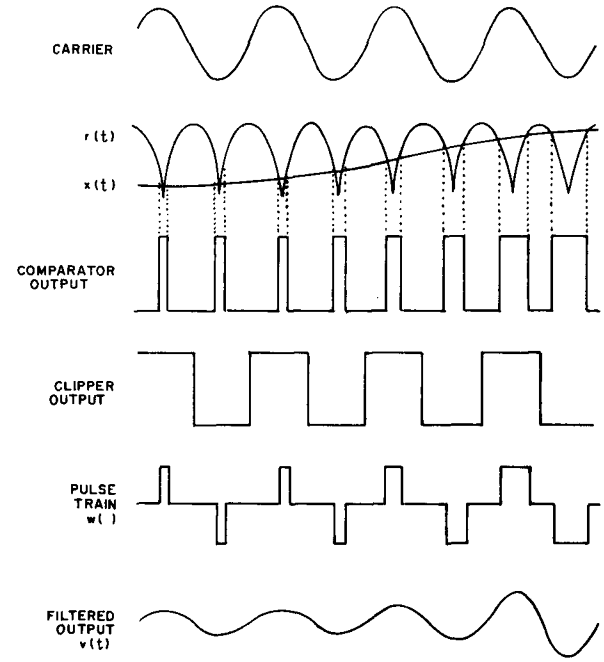


Fig. 4. Waveforms in class-D AM system.

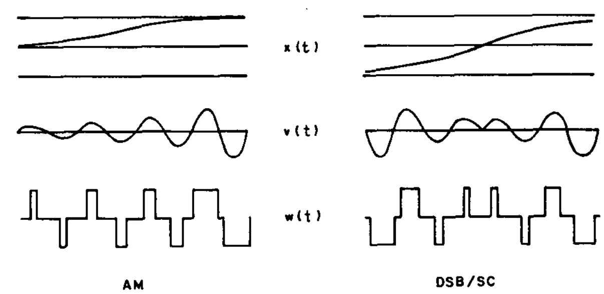


Fig. 5. Generation of AM and DSB/SC.

Generation of double-sideband suppressed-carrier (DSB/SC) signals can be accomplished with some simple additions to the basic AM transmitter. Fig. 5 illustrates the phase shift or polarity reversal in a DSB/SC signal. This requires the RF PWM transmitter to reverse its pulse polarity. The only additions required to the AM transmitter are a device to produce the absolute value of $x(t)$ for use in the comparator and logic to reverse pulse polarity when $x(t)$ is negative.

It is also possible to generate single-sideband suppressed-carrier (SSB/SC) signals, but the apparatus is again more complicated. Since any signal can be regarded as a single-sideband signal, transmission of ISB, VSB, and other complicated signals is also possible.

The phasing method may be used to employ RF PWM as a single-sideband modulator. The phasing method requires that the audio $x_q(t)$ modulates a phase-shifted carrier $\cos \omega_c t$ and the phase-shifted (±90°) audio $x_p(t)$ modulates the carrier $\sin \omega_c t$. The sum of the two modulated signals produces single sideband. When this principle is used with RF PWM, a phase-shifted carrier and $x_q(t)$ are applied to one RF pulse-width modulator, while the carrier and $x_p(t)$ are applied to

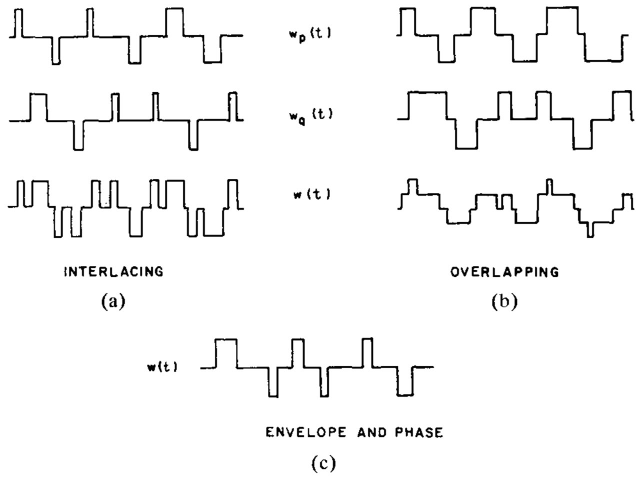
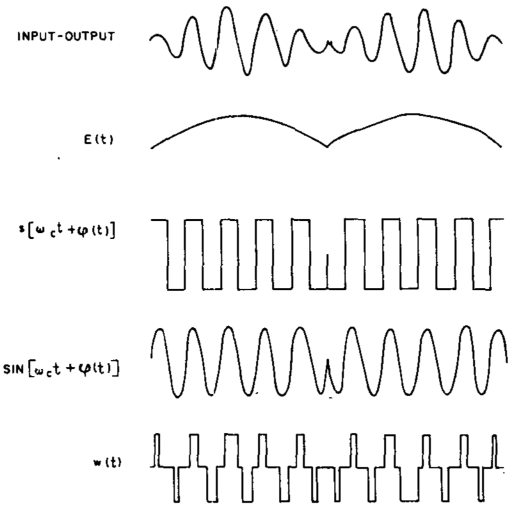


Fig. 6. Class-D SSB techniques.



the other RF pulsewidth modulator (Fig. 6). There are then two methods of combining the outputs of the two modulators. The interlacing method restricts $y(t)$ in both modulators to a maximum of $\pi/4$ and gates all pulses to the output amplifiers. The overlapping method restricts $y(t)$ to the normal maximum $\pi/2$, and uses logic to generate double (± 2) or 0 height pulses when the pulse outputs from the two modulators overlap.

Either the filter or phasing method of generating SSB can be used with RF PWM acting as an amplifier. This method requires the generation of a low-level SSB signal, and uses Kahn's envelope elimination and restoration techniques [8], [9]. A single-sideband signal can be represented by either in-phase and quadrature components or by an envelope and phase:

$$v(t) = x_p(t) \sin \omega_c t + x_q(t) \cos \omega_c t = E(t) \sin [\omega_c t + \varphi(t)]. \quad (1.5)$$

The relations between these two descriptions are

$$E(t) = \sqrt{x_p^2(t) + x_q^2(t)} \quad (1.6)$$

and

$$\varphi(t) = \arctan \left[\frac{x_q(t)}{x_p(t)} \right]. \quad (1.7)$$

(Note that the inverse tangent function must place $\varphi(t)$ in the proper quadrant.)

The envelope signal $E(t)$ is produced by an envelope detector (Fig. 7) and replaces $x(t)$ as the audio input to an AF RF PWM transmitter. A clipper removes amplitude information and produces a phase-shifting square wave. A bandpass filter produces the fundamental frequency component of the square wave, which then becomes the RF input to the AM transmitter (in actual implementation, the clipper and filter need not be duplicated in the AM transmitter). The reference wave then carries the phase information, which then appears as pulse position modulation. Recombination with the envelope information provided by pulsewidth modulation produces the desired SSB signal.

II. Amplifier Operation and Efficiency

A width-modulated class-D RF amplifier can be implemented by a set of four transistors connected to the load by a series-tuned circuit (Fig. 8). For monopolar operation, transistors $Q2$ and $Q3$ may be replaced by diodes, and $Q4$ is not required. For convenience in subsequent analyses, the supply voltages

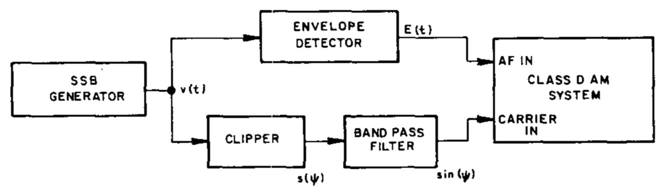


Fig. 7. Class-D SSB generation using Kahn's method.

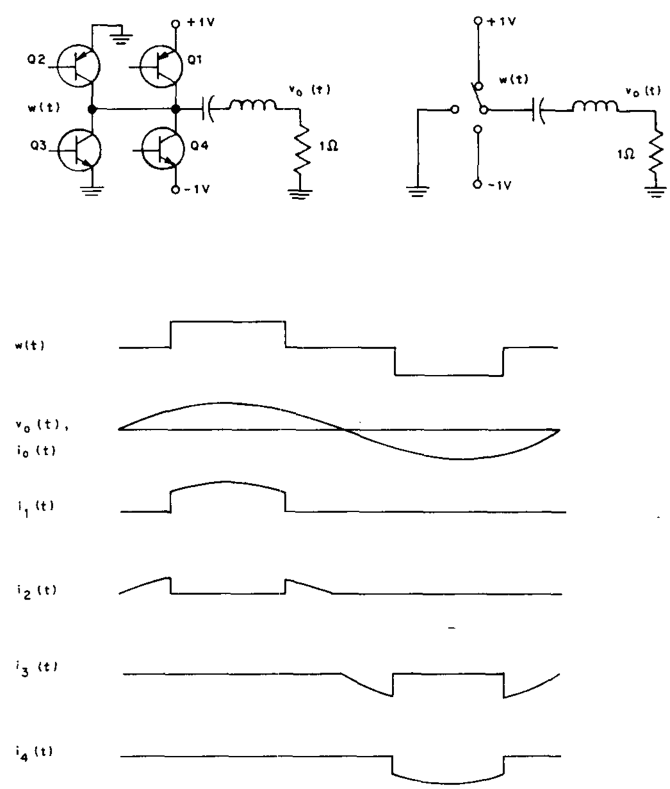


Fig. 8. Class-D RF amplifier circuit. Currents i_1–i_4 represent the current Flow in transistor Q_1–Q_4, respectively.

have been normalized to ± 1 V and the load has been normalized to 1 Ω.

To understand the basic amplifier operation, first assume that switching occurs instantaneously and saturation voltages are zero. The set of transistors acts as a three-position switch to apply a rectangular voltage waveform to the tuned circuit. The tuned circuit presents very low impedance to the fundamental frequency (carrier frequency) component of the voltage waveform, allowing current to flow into the load at that frequency. However, harmonics of the carrier see a very high impedance, hence negligible harmonic current is allowed to flow. Note that a ground connection must be made (when neither +1 nor −1 connections are made) so that the current flowing in the load can remain sinusoidal.

The voltage appearing on the load is the fundamental component of the pulse train. If

$$v_0(\omega t) = i_0(\omega t) = b \sin \omega t, \tag{2.1}$$

the amplitude b is given by

$$b = \frac{4}{\pi} \sin y, \tag{2.2}$$

where y is the pulse half-width in radians. The power output for a given pulsewidth is then

$$P_0 = \frac{b^2}{2} = \frac{8}{\pi^2} \sin^2 y. \tag{2.3}$$

Ideal switches consume no power, hence an ideal class-D amplifier has an efficiency of 100 percent. In practice; however, neither switching time nor saturation voltages are negligible and efficiency is reduced.

A reasonably accurate assessment of the effects of non-instantaneous switching and nonzero saturation voltages can be obtained by making three simplifying assumptions:

1) the output current and voltage are essentially those of an ideal amplifier;
2) current and voltage transitions can be modeled as linear (ramp) shapes;
3) output current remains constant during switching.

Let a transition time be specified by τ, which is in radians (if a transition occupies 1 percent of an RF cycle $\tau = 0.01 \cdot 2\pi$). Also let

$$\theta = \omega_c t \tag{2.4}$$

for convenience. The power dissipated in one periodic transition by one transistor is determined by integrating the voltage–current product (Fig. 9)

$$P_{DR\ \mathrm{trans}} = \frac{1}{2\pi}\int_{\theta}^{\theta+\tau} v_q(\theta')\, i_q(\theta')\, d\theta' \tag{2.5}$$

$$= \frac{|i_q(\theta)|}{2\pi} \int_{\theta}^{\tau} \left(1 - \frac{\theta}{\tau}\right)\left(\frac{\theta}{\tau}\right) d\theta \tag{2.6}$$

$$= \frac{1}{2\pi}\,\frac{\tau}{6}\, |i_q(\theta)|. \tag{2.7}$$

The pulsewidth determines both the peak value of the output current and the point at which the transition occurs, so

$$P_{DR\,\mathrm{trans}} = \frac{1}{2\pi} \cdot \frac{\tau}{6} \cdot \frac{4}{\pi} \sin y \cdot \cos y \tag{2.8}$$

$$= \frac{\tau}{6\pi^2} |\sin 2y|. \tag{2.9}$$

There are two transistors switching at each transition. If σ represents the total time spent in all transitions, then the total power dissipated in switching is given by

$$P_{DR} = \frac{\sigma}{3\pi^2} |\sin 2y|. \tag{2.10}$$

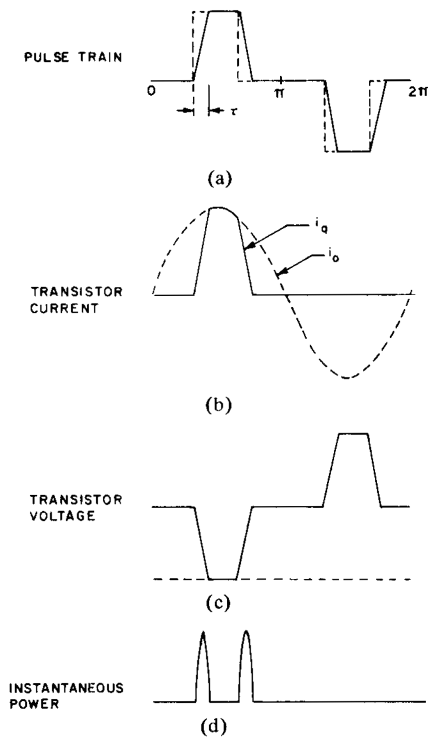


Fig. 9. Waveforms determining P_D.

Let v_s be a normalized saturation voltage (i.e., if the actual supply voltage is 10 V and the actual saturation voltage is 0.5 V, $v_s = .05$).

Insert sources of voltage v_s in series with the ideal switch such that the current flow through them causes power to be dissipated (Fig. 10).

Since only one transistor conducts current at a time, only one integration is required:

$$P_{DS} = \frac{1}{2\pi}\int_0^{2\pi} v_s |i_q(\theta)|\, d\theta \tag{2.11}$$

$$= \frac{8}{\pi^2} v_s \sin y. \tag{2.12}$$

The input power is then the sum of the output power and the dissipated power, so the efficiency is determined by

$$\eta = \frac{P_0}{P_i} = \frac{P_0}{P_0 + P_{DR} + P_{DS}}. \tag{2.13}$$

Efficiency as a function of output signal amplitude b is plotted in Fig. 11, with v_s and σ as parameters.

The efficiency of a monopolar amplifier can be found without modification of the previous equations. First, note that the output amplitude is halved since only one pulse per cycle contributes to the carrier. Second, σ will be smaller, since output current is halved. The efficiency of a monopolar amplifier thus may be determined from Fig. 11 by using an appropriate value of σ and restricting the output amplitude so that

$$b \leqslant 2/\pi. \tag{2.14}$$

At high frequencies, where σ is large, it may be advantageous to use monopolar RF PWM to reduce the switching losses.

An interesting and important aspect of the efficiency curves of Fig. 11 is that for values of $\sigma/2\pi$ and v_s as large as 0.1 (10 percent), efficiency exceeds that of class B (straight line

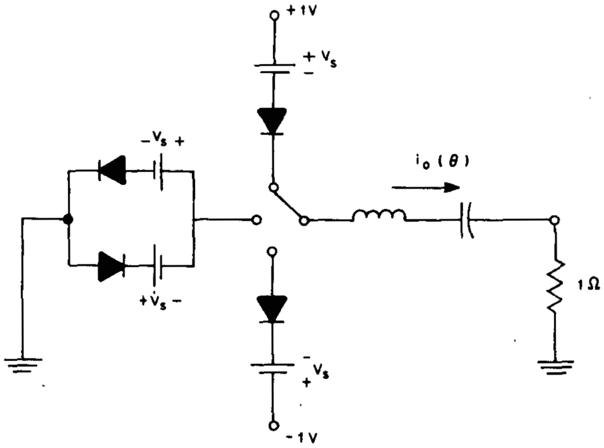


Fig. 10. Saturation voltage effects.

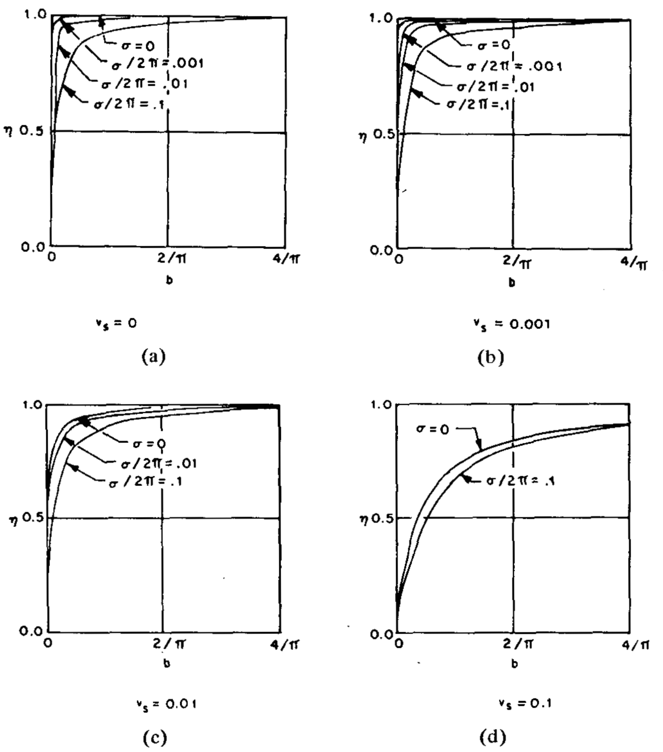


Fig. 11. Efficiency of a class-D RF amplifier.

from 0 at $b = 0$ to 0.785 at maximum output) for all output amplitudes. This requires a switching speed of approximately 20 times the carrier frequency and a saturation voltage of one-tenth the supply voltage. The efficiency of the RF PWM amplifier also exceeds that of an AF PWM amplifier (producing the same frequency output). Not only is the switching frequency reduced, but transitions occur at low current points when output current is large.

III. Spectral Characteristics

The frequency characteristics determine, to a great extent, the usefulness of an amplifier. Unwanted frequency components produced by an RF amplifier can be classified as intermodulation (distortion) products that occur near the carrier frequency, and spurious products that can occur at any frequency.

The spectrum of a width-modulated pulse train can be determined using a two-dimensional Fourier series [10]. First, an ordinary Fourier series for the basic pulse train $w(t)$ is determined with width and position held as constant parameters. This results in each Fourier coefficient depending upon parameters y and φ. When y and φ become time-varying functions $y(t)$ and $\varphi(t)$, each Fourier coefficient also becomes a time-varying function. An ordinary Fourier series can be determined for each modulation function. These spectra then modulate their respective components of the basic pulse train.

The basic series for the rectangular pulse train shown in Fig. 12 are as follows.

Monopolar:

$$w(t;y,\varphi) = \frac{y}{\pi} + \frac{2}{\pi} \sum_{m=1}^{\infty} \left[\frac{(-1)^m}{2m} \sin 2my \cos 2m(\omega_c t + \varphi) + \frac{(-1)^{m+1}}{2m-1} \sin (2m-1)y \sin (2m-1)(\omega_c t + \varphi) \right] \quad (3.1)$$

Bipolar:

$$w(t;y,\varphi) = \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{2m-1} \sin (2m-1)y \sin (2m-1)(\omega_c t + \varphi). \quad (3.2)$$

Note that the bipolar pulse train contains only odd harmonics, and the odd harmonics are the same (except for a factor of $\frac{1}{2}$ in both. A special waveform, which will be required, is the clipped sine wave $s(\theta)$:

$$s(\theta) = \operatorname{sgn} (\sin \theta) = \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{1}{2m-1} \sin (2m-1)\theta. \quad (3.3)$$

Fourier coefficients derived will be specified by both waveform and order. For example, b_{x2} indicates the coefficient of the $\sin 2\theta$ component of $x(\theta)$.

The argument of functions depending on the modulating signal $x(t)$ will be denoted by

$$\theta = \omega_x t = 2\pi f_x t. \quad (3.4)$$

For simplicity; θ_c, f_c, and ω_c similarly represent the RF (carrier) frequency.

Amplitude modulation (AM) and DSB/SC modulating signals (with 100 percent modulation) are represented by

$$x(\theta) = \tfrac{1}{2} + \tfrac{1}{2} \sin \theta \quad (3.5)$$

and

$$x(\theta) = \sin \theta. \quad (3.6)$$

The latter is equivalent to a two-tone SSB signal, commonly used as a test signal (f_x becomes half the tone separation). It is also a convenient signal to use in the analysis of RF PWM, since modulation functions can be evaluated without extreme difficulty.

For DSB/SC,

$$E(\theta) = |x(\theta)| = |\sin \theta| = \sin \theta \, s(\theta) \quad (3.7)$$

and

$$\varphi(\theta) = \pi \left[\frac{1 - s(\theta)}{2} \right] \quad (3.8)$$

Since the phase shift is either 0 or π for all DSB/SC (or AM) signals, any effects of phase shifting of a harmonic can

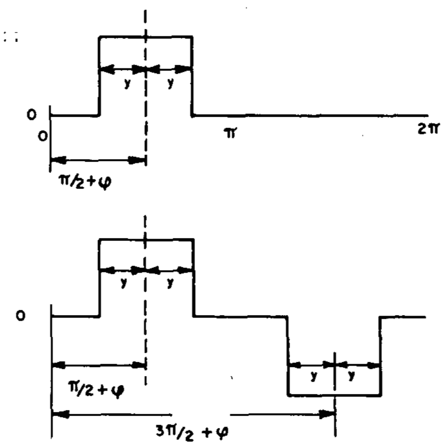


Fig. 12. Basic pulse train.

be reduced to a polarity reversal of that harmonic. This polarity reversal can be combined with amplitude modulation of the particular harmonic to form a modulation function $z_k(\theta)$ This allows writing the modulated bipolar pulse train as

$$w(\theta) = \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1} z_{2m-1}(\theta)}{2m-1} \sin \omega_c t. \tag{3.9}$$

The inverse sine relationship between the modulating signal and pulsewidth causes the pulsewidth for a DSB/SC to be composed of pieces of ramp waveforms:

$$y(\theta) = \arcsin |\sin \theta| \tag{3.10}$$

$$= \frac{\pi}{2} \Lambda(\theta) s(\theta), \tag{3.11}$$

where $\Lambda(\theta)$ is a unit height triangular wave (Fig. 13). (This can be seen by evaluation of $y(\theta)$ for each $\pi/2$ interval of θ.)

The phase shifts of a DSB/SC single-tone signal cause a polarity reversal of the odd harmonics since all even multiples of π produce no additional effect:

$$\sin (2m-1)[\omega_c t + \varphi(\theta)] = \sin [(2m-1)\omega_c t + (2m-1)\varphi(\theta)] \tag{3.12}$$

$$= \sin [(2m-1)\omega_c t - \varphi(\theta)] \tag{3.13}$$

$$= s(\theta) \sin (2m-1)\omega_c t. \tag{3.14}$$

The amplitude variations are similarly analyzed by piecewise evaluation over $\pi/2$ intervals of θ (Fig. 13). Noting that a sine of a ramp waveform must produce a piece of a sine or cosine function:

$$\sin (2m-1) y(\theta) = \sin (2m-1)\frac{\pi}{2}\Lambda(\theta) s(\theta) \tag{3.15}$$

$$= s(\theta) \sin [(2m-1)\frac{\pi}{2}\Lambda(\theta)] \tag{3.16}$$

$$= s(\theta) \sin (2m-1)\theta. \tag{3.17}$$

When this is combined with the polarity reversal caused by phase shifting,

$$z_{2m-1}(\theta) = \sin (2m-1)\theta\, s(\theta) s(\theta) \tag{3.18}$$

$$= \sin (2m-1)\theta. \tag{3.19}$$

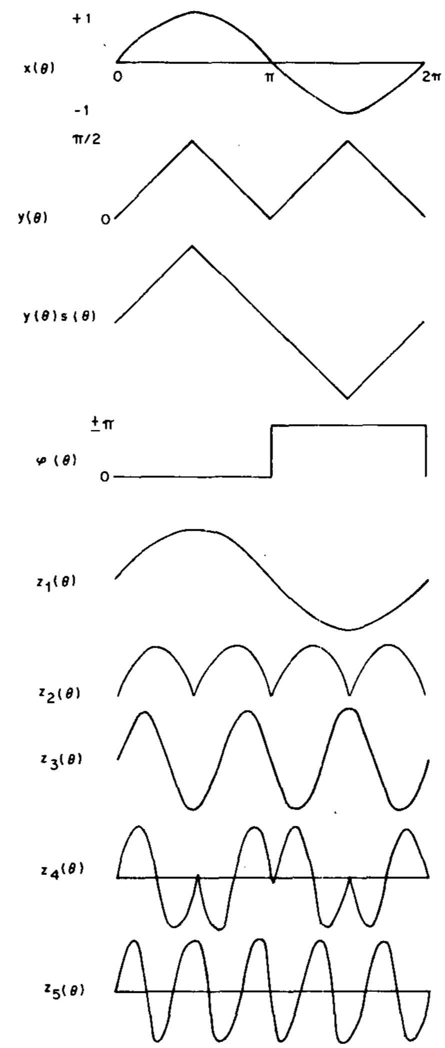


Fig. 13. Waveforms generated by DSB/SC.

The modulation of the kth odd harmonic of the carrier is thus a simple sinusoid of k times the audio frequency of the carrier modulation. Thus the spurious products generated are all strictly band limited around the odd harmonics in a bipolar RF pulsewidth modulator. The modulation frequency can be as high as one-half of the carrier frequency before the spurious products cross into the passband.

This important result can be generalized to apply to all double-sideband (including AM) modulating signals. For any double signal, $\varphi(\theta)$ is either 0 or $\pm\pi$, according to

$$\varphi(\theta) = \pi \left[\frac{1 - \operatorname{sgn} x(\theta)}{2}\right], \tag{3.20}$$

so

$$\sin [\omega_c t + \varphi(\theta)] = \operatorname{sgn} x(\theta) \sin \omega_c t. \tag{3.21}$$

The amplitude variation of an odd harmonic can be expanded using a series given by Jolley [11]:

$$\sin [k \arcsin E] = kE - \frac{k(k^2 - 1^2)}{3!} E^3 + \frac{k(k^2 - 1^2)(k^2 - 3^2)}{5!} E^5 - \cdots. \tag{3.22}$$

Since k is odd, all terms containing E^m where $m > k$ also contain the factor $(k^2 - k^2)$. As a result, (3.22) terminates with the term containing E^k. The modulation function is now described by

$$z_k(\theta) = \sin\left[k \arcsin E(\theta)\right] \operatorname{sgn} x(\theta) \tag{3.23}$$

$$= kE(\theta) \operatorname{sgn} x(\theta) - \frac{k(k^2 - 1^2)}{3!} E^3(\theta) \operatorname{sgn} x(\theta) + \cdots$$
$$\pm \frac{k(k^2 - 1^2) \cdots [k^2 - (k-2)^2]}{k!} E^k(\theta) \operatorname{sgn} x(\theta) \tag{3.24}$$

$$= kx(\theta) - \frac{k(k^2 - 1^2)}{3!} x^3(\theta) + \cdots$$
$$\pm \frac{k(k^2 - 1^2) \cdots [k^2 - (k-2)^2]}{k!} x^k(\theta), \tag{3.25}$$

since $\operatorname{sgn} x(\theta)$ restores the polarity to $E(\theta)$. Thus the modulation of the kth odd harmonic of the carrier is composed of a sum of odd powers of the basic modulation, not exceeding the kth power.

The spectrum of the product of two time functions can be determined by convolution of the spectrum of each. For example, the spectrum $X_2(\omega)$ of $x^2(\theta)$ can be found by convolution of $X(\omega)$ with itself:

$$X_2(\omega) = X(\omega) * X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(u) X(\omega - u)\, du. \tag{3.26}$$

If $x(\omega)$ is band limited so that

$$X(\omega) = 0, \qquad |\omega| \geqslant \omega_x, \tag{3.27}$$

it is apparent a nonzero product can occur in (3.26) only for frequencies less than $2\omega_x$. Thus

$$X_2(\omega) = 0, \qquad |\omega| \geqslant 2\omega_x. \tag{3.28}$$

By repetition of the above process,

$$x_k(\omega) = 0, \qquad |\omega| \geqslant k\omega_x. \tag{3.29}$$

Thus any frequencies contained in $z_k(\theta)$ can be no higher than kf_x, and the spurious products are band limited.

This characteristic of RF PWM has been verified by digital computer simulation and by observation of a prototype amplifier [12]. Results of the computer simulations for DSB/SC and AM are shown in Figs. 14 and 15. A normalized carrier frequency of 10 and normalized audio frequency of 1 were used. Pulse transitions were computed by finding iteratively the intersection of $|x(t)|$ and $r(t)$, as would be done by the comparator of a real amplifier. Fourier series were calculated directly from the pulse train (not from modulation functions). The vertical axis is the absolute magnitude of a frequency component, calculated by

$$c_{wf} = \sqrt{a^2{}_{wf} + b^2{}_{wf}}\ . \tag{3.30}$$

The band limited characteristics of bipolar RF PWM used for double-sideband signals make it highly desirable for RF amplification. Spurious products are band limited and separated from the desired output, and hence are easily filtered out. This is not possible, for example, with AF PWM, where infinite-bandwidth spurious products are inherent by-products of the modulation process.

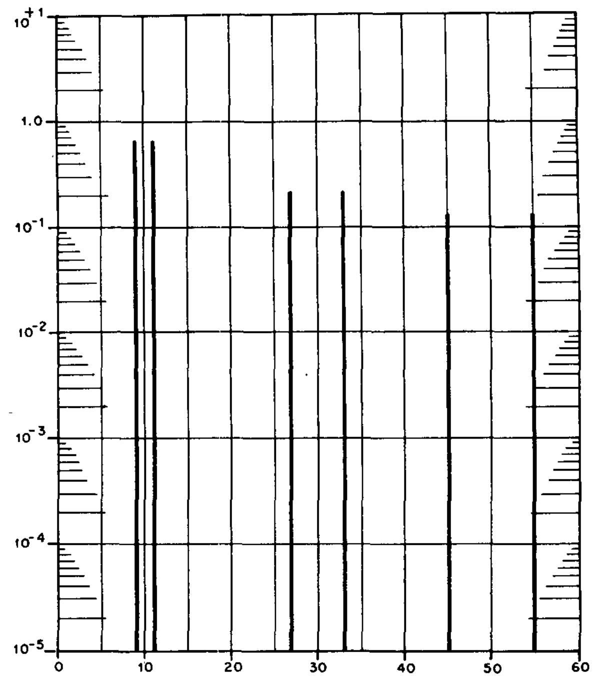


Fig. 14. Simulation of bipolar DSB/SC spectrum.

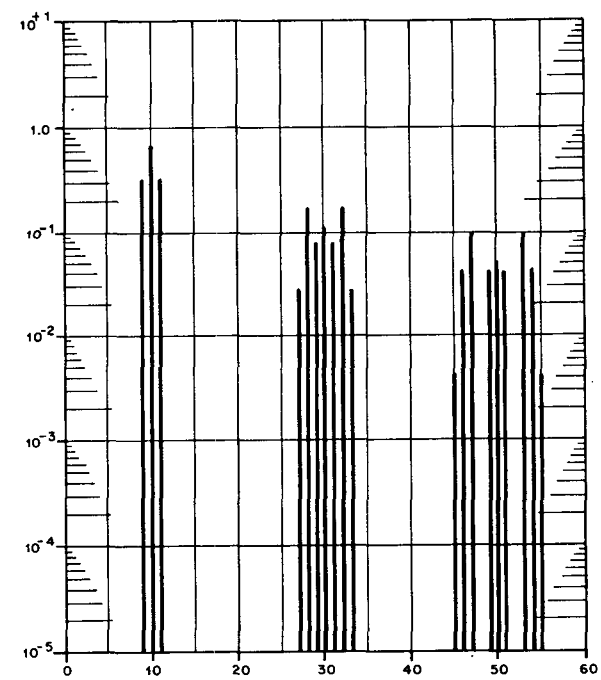


Fig. 15. Simulation of bipolar AM spectrum.

Unfortunately, RF PWM generation of single-sideband signals using Kahn's method does not have band-limited odd harmonics. Some general properties of the modulation functions have been determined [12], but exact evaluation is difficult. The phase must be a continuous function (a jump from 0 to 2π is not really a discontinuity for phase), except for abrupt $\pm\pi$ jumps at zero output amplitude (DSB/SC). However, the derivative $d\varphi(\theta)/d\theta$ may have abrupt jumps. A consequence of this is a spectrum of $z_k(\theta)$ that falls off as $1/f^2$ from

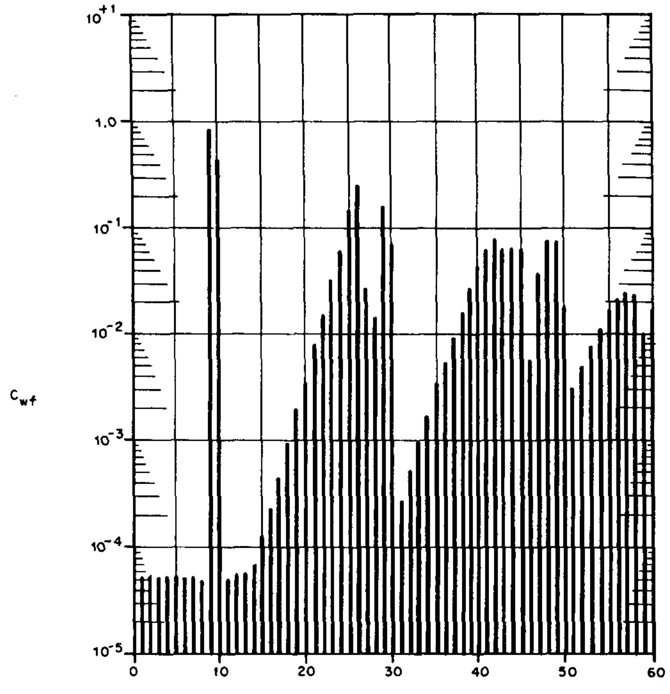


Fig. 16. Simulation of bipolar compatible AM spectrum.

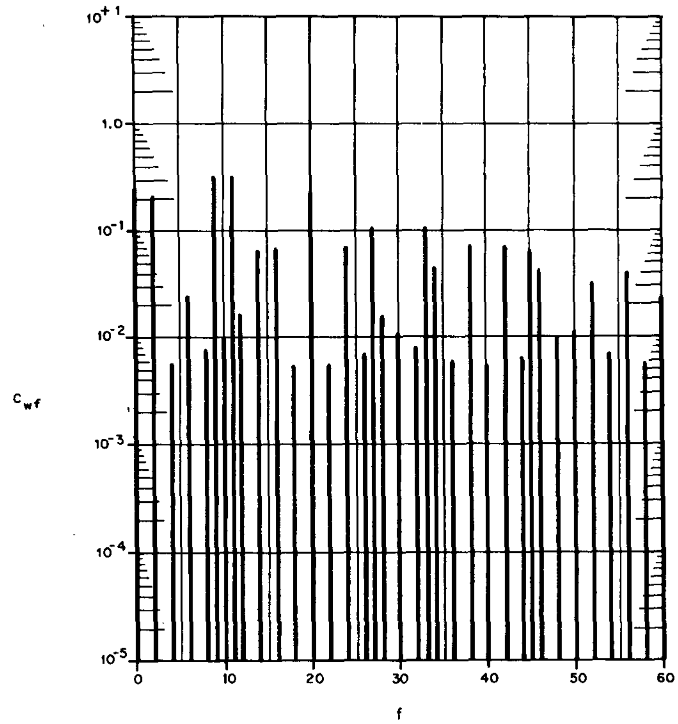


Fig. 17. Simulation of monopolar DSB/SC spectrum.

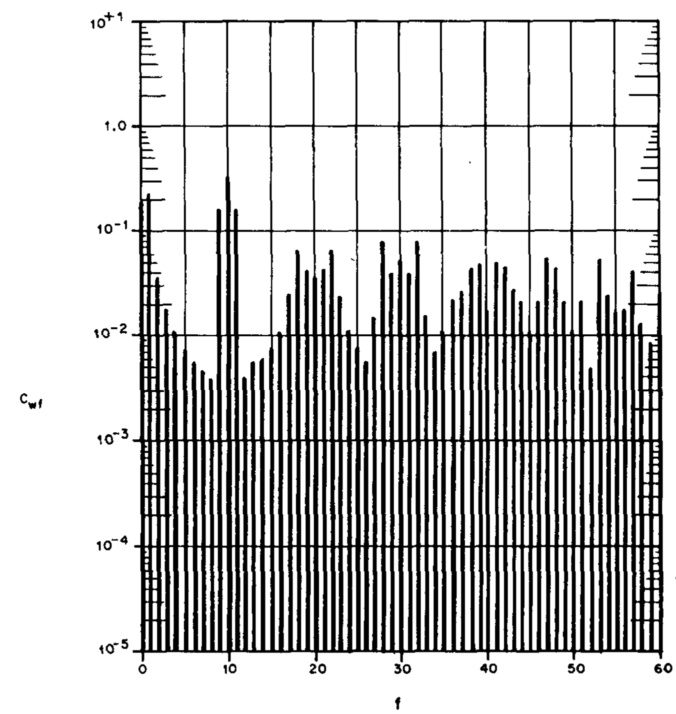


Fig. 18. Simulation of monopolar AM spectrum.

kf_c [13]. While this does not specify a maximum value of spurious products, the 40-dB/decade (AF) decay of spurious products should be adequate in most cases. This has also been verified for several signals. The spectrum of a compatible AM (USB and carrier) is shown in Fig. 16. Spurious products are down by 70 dB for $f_c/f_x = 10$. For actual operating frequencies, spurious products should be negligible. (The spurious products below $f = 12$ in Fig. 16 are believed to be computer errors. The pulse-finding program for SSB signals is different from the one for DSB signals, and produces similar errors when used with a DSB/SC signal). An interesting property of SSB signals is that reversal of the sidebands of the desired signal also reverses the sidebands of the odd harmonics. It appears that for practical operating frequencies, spurious products produced by SSB signals by bipolar RF PWM will be negligible.

Monopolar RF PWM is not only slightly easier to implement than bipolar RF PWM, but can have improved efficiency if transition times are significant. Even harmonics can also appear as a result of unequal positive and negative voltage supplies. Thus knowledge of the spectra of monopolar RF PWM is desirable.

The odd harmonics produced by monopolar RF PWM are identical to those produced by bipolar RF PWM, except for amplitude. Properties of even harmonics can be determined by the same methods used on the odd harmonics.

For double-sideband signals, a phase shift of π at f_c produces a phase shift of an even multiple of π at an even harmonic of f_c. Thus

$$\sin 2m[\omega_c t + \varphi(\theta)] = \sin 2m\omega_c t; \tag{3.31}$$

the phase information disappears.

The series (3.22) applies equally well to an even harmonic. However, with k even, a factor of $(k^2 - k^2)$ does not occur, and the series is infinite. Infinite bandwidth even modulating functions are, therefore, generally expected. Computer simulations of monopolar DSB/SC and AM spectra are shown in Figs. 17 and 18.

The nature of $z_{2m}(\theta)$ for single-tone DSB/SC can be determined by piecewise evaluation of the waveform (Fig. 13).

$$z_{2m} = \sin [2m \arcsin |\sin \theta|] \tag{3.32}$$

$$= \sin 2m\theta \, s(2\theta). \tag{3.33}$$

The modulation of the kth even harmonic is thus seen to be a sinusoid of k times the modulating frequency experiencing phase (or polarity) reversals at twice the modulating frequency. Since the spectrum of the square wave $s(2\theta)$ is known, the spectrum of $z_{2m}(\theta)$ contains even-order cosine terms.

If

$$z_k(\theta) = a_{z_k0} + \sum_{m=2,4,6,\cdots} a_{z_km} \cos m\theta \tag{3.34}$$

represents the series, then for large m,

$$|a_{z_km}| \approx \frac{8}{\pi} \left| \frac{k}{m^2 - k^2} \right|. \tag{3.35}$$

Given the carrier and audio frequencies, it is now possible to determine the maximum spurious product due to a given harmonic of the carrier falling on a particular frequency. In fact, a worst case product (assuming all spurious products add in phase) can be determined [12]. For monopolar RF PWM, the worst possible product occurring near f_c is

$$D = 2\frac{f_x^2}{f_c^2}. \tag{3.36}$$

For $f_c/f_x \geqslant 250$, D is 80 dB or more below either of the two tones, so for ordinary high-frequency transmitters, monopolar RF PWM should be satisfactory. However, for medium- and low-frequency transmitters (e.g., a broadcast transmitter at 1.25 MHz or less), bipolar RF PWM would be required.

IV. Comments and Conclusions

Since the pulse waveform itself is used to generate the modulation of the carrier, any distortion of the pulse waveform can introduce distortion of the desired signal. Effects that cause distortion include saturation voltage, pulse bias, and unequal rise and fall times.

Saturation voltage introduces a low-level clipped signal. Pulse bias (elongation or shortening) causes a deviation from a linear transfer characteristic with an abrupt jump when crossing zero. Pulse rise and fall time contribute a similar effect, but if rise and fall times are equal, there is negligible distortion. These effects can be analyzed by considering the pulse distortion as a train of small pulses added to the train of "ideal" pulses. The added distortion pulses are position modulated by width of ideal pulses. The largest distortion product for DSB/SC is roughly proportional to the total bias error (i.e., if the net error is 1 percent, the IMD product is approximately 40 dB below the desired signal). For AM signals, the products are slightly smaller. Analytical results are verified in [12].

A prototype 3-W AM transmitter was built to verify the concept qualitatively. The spectrum of this amplifier is shown in Fig. 19, and compares favorably with the simulation of Fig. 15. Equipment and resources available limited the carrier frequency to 100 kHz. Higher frequency operation should be possible, since unmodulated class-D RF amplifiers with carrier frequencies as high as 60 MHz have been built [3].

Distorting effects of pulse bias and unequal transition times can be reduced by the use of envelope feedback. The effect of a feedback loop is to set the pulsewidth to the correct width in spite of errors in pulse shape. Some preliminary results indicate that 20–30 dB reduction in intermodulation distortion products can be obtained. This allows use of the width-modulated class-D RF amplifier at high frequencies where efficiency is still good but open loop distortion is too high.

Previous high-efficiency methods of obtaining a modulated RF carrier have involved a constant-width class-D RF amplifier that is collector modulated by a pulsewidth modulation audio amplifier. In addition to the simplicity offered by eliminating the circuitry of the collector modulator, the width-modulated amplifier has an audio frequency response that is limited only by the RF output filter. The spurious products generated by the width-modulated class-D RF amplifier should also be far enough from the carrier that they can be removed easily. Due to the improved efficiency, relatively small transistors can be used. Thus class D may be a good way to extend solid state to higher power and higher frequency applications.

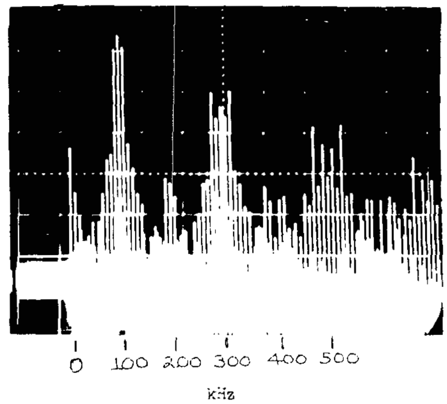


Fig. 19. Spectrum of pulse train of prototype amplifier.

Acknowledgment

The author wishes to thank Prof. P. R. Bond for his advice and assistance. He also wishes to thank Iowa State University for making available computer time that was used for numerical evaluations and simulations required to obtain and verify the results. The author also wishes to thank Cincinnati Electronic Corporation for its assistance in the preparation of this manuscript.

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